MOL-MODE ANALYSIS WITH PRECISE RESOLUTION BY AN ENHANCED AND GENERALIZED LINE ALGORITHM

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ABSTRACT

A new algorithm based on the Method of Lines for computation of eigenmodes in waveguides for microwave and millimeter frequencies is proposed and substantiated. The algorithm uses discretization lines in different directions and in some regions of the guide cross-section in two perpendicular directions. This enables a precise computation of the fields and propagation parameters. The algorithm allows analysis of complex waveguide structures.

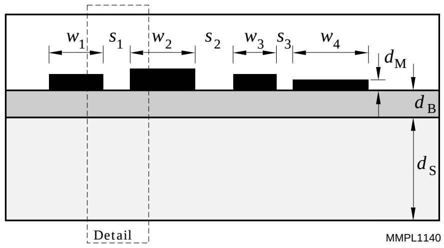

Fig. 1: Cross-section of a millimeter-wave circuit

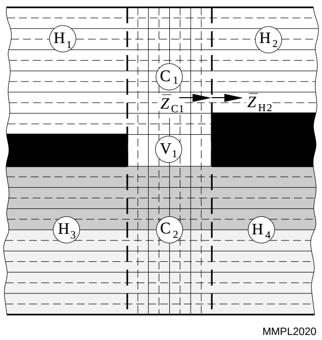

Fig. 2: Part of cross-section in Fig. 1 with details of the discretization

INTRODUCTION

Modern integrated circuits for microwave and millimeter frequencies consist of a number of composite layers with embedded metallic strips. The trend toward miniatuarization results in closer positioning of all elements. Usually the dimensions of the waveguide cross-sections are small compared with the length of sections in the circuits. To describe the circuit behavior correctly it is necessary to precisely determine the eigenmodes and especially the propagation constant. Fig. 1 gives an example of a complex waveguide cross section. Such a waveguide structure is important, e.g. for Lange-couplers in millimeter wave technology. In this case the metallic strips may have equal dimensions. Generally the dimensions in the cross-section are different. The slot widths between the strips and their heights may be small compared with the widths of the metallic strips and the thickness of the layers. To analyze such waveguide structures precisely, the modeling algorithm must take these facts into account. Hence in this contribution an adequate new procedure is described based on the Method of Lines (MoL). In the MoL we discretize the field not completely but only as long as necessary. In the remaining direction the field dependence is calculated analytically. In Fig. 2 it is shown how the new discretization should be performed in the given example of the structure. We have some regions which are bounded by metallizations on two sides. In these regions we use discretization lines parallel to the metallization surface. In the regions $H_i (i = 1, 2, 3, 4)$ we have horizontal and in region V_1 vertical discretization lines. The regions C_1 and C_2 connect the regions with horizontal and vertical discretization lines. Therefore, in these regions the lines from the neighboring regions should have a continuation. This in turn means the use of crossed lines in regions C_1 and C_2. The numbers of discretization lines in horizontal and vertical directions can be chosen separately. Therefore, the number of lines, e.g. between the strips, should be chosen large enough to obtain a high accuracy without increasing the numerical effort appreciably. In the regions with unidirectional discretization lines the field description is achived as in [1][2].

GENERAL PORT RELATION

The field description in the region with crossed lines can be achieved in the following way: The field outside R can be uniquely determined if the tangential fields are known at the surface of R (uniqueness theorem), that means at the ports A, B, C and D. In view of linearity of all materials and the Maxwell equations, the following relation in matrix form between the tangential fields at the ports A, B, C and D of the general region R holds

$$\begin{bmatrix} \mathbf{H}_{\mathrm{A}} \\ -\mathbf{H}_{\mathrm{B}} \\ \mathbf{H}_{\mathrm{C}} \\ -\mathbf{H}_{\mathrm{D}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{y}_{\mathrm{AA}} & \cdots & \boldsymbol{y}_{\mathrm{AD}} \\ \vdots & & \vdots \\ \boldsymbol{y}_{\mathrm{DA}} & \cdots & \boldsymbol{y}_{\mathrm{DD}} \end{bmatrix} \begin{bmatrix} \mathbf{E}_{\mathrm{A}} \\ \mathbf{E}_{\mathrm{B}} \\ \mathbf{E}_{\mathrm{C}} \\ \mathbf{E}_{\mathrm{D}} \end{bmatrix} \tag{1}$$

$\mathbf{E}_{\mathrm{U}}$ and $\mathbf{H}_{\mathrm{U}}$ (U = A, B, C, D) are supervectors of the discretized tangential fields at port U. Each supervector consists of two vectors for the two tangential components. In more compact form we can write

$$\begin{bmatrix} \widehat{\mathbf{H}}_{\mathrm{AB}} \\ \widehat{\mathbf{H}}_{\mathrm{CD}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{y}_{\mathrm{AB}}^{\mathrm{AB}} & \boldsymbol{y}_{\mathrm{AB}}^{\mathrm{CD}} \\ \boldsymbol{y}_{\mathrm{CD}}^{\mathrm{AB}} & \boldsymbol{y}_{\mathrm{CD}}^{\mathrm{CD}} \end{bmatrix} \begin{bmatrix} \mathbf{E}_{\mathrm{AB}} \\ \mathbf{E}_{\mathrm{CD}} \end{bmatrix} \tag{2}$$

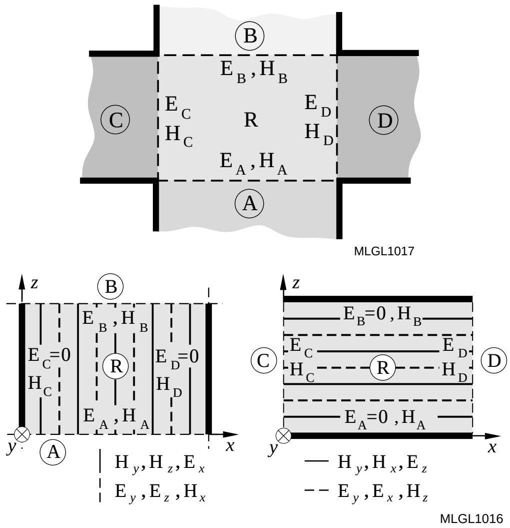

Fig. 3: Field calculation in a region R using crossed lines:
a) general region R with definitions
b) ports C and D short-circuited: analysis using vertical lines
c) ports A and B short-circuited: analysis using horizontal lines

$$\begin{aligned} \widehat{\mathbf{H}}_{\mathrm{AB}} &= \left[\mathbf{H}_{\mathrm{A}}^t, -\mathbf{H}_{\mathrm{B}}^t\right]^t & \widehat{\mathbf{E}}_{\mathrm{AB}} &= \left[\mathbf{E}_{\mathrm{A}}^t, \mathbf{E}_{\mathrm{B}}^t\right]^t \\ \widehat{\mathbf{H}}_{\mathrm{CD}} &= \left[\mathbf{H}_{\mathrm{C}}^t, -\mathbf{H}_{\mathrm{D}}^t\right]^t & \widehat{\mathbf{E}}_{\mathrm{CD}} &= \left[\mathbf{E}_{\mathrm{C}}^t, \mathbf{E}_{\mathrm{D}}^t\right]^t \end{aligned} \tag{3}$$

The admittance matrices can be calculated in the following way: By short-circuiting the ports C and D (using metallic walls) we obtain magnetic field parts at the ports A and B and even at ports C and D from the electric field $\mathbf{E}_{\mathrm{AB}}$. All these field parts are determined using vertical discretization lines parallel to the metallic side walls. From these partial fields the matrices $\boldsymbol{y}_{\mathrm{AB}}^{\mathrm{AB}}$ and $\boldsymbol{y}_{\mathrm{CD}}^{\mathrm{AB}}$ are obtained. Similarly short-circuiting ports A and B and using horizontal discretization lines we obtain the matrices $\boldsymbol{y}_{\mathrm{AB}}^{\mathrm{CD}}$ and $\boldsymbol{y}_{\mathrm{CD}}^{\mathrm{CD}}$. For the regions with unidirectional discretization lines analogous equations as above can be written [1].

ADMITTANCE MATRICES

In this subsection some details for the determination of the matrices in eq. (2) will be given. As example we describe the determination of the supermatrices $\widehat{\overline{\boldsymbol{y}}}_{\mathrm{AB}}^{\mathrm{AB}}$ and $\widehat{\overline{\boldsymbol{y}}}_{\mathrm{CD}}^{\mathrm{AB}}$ in transform domain. For this purpose the ports C and D are short circuited. The discretization lines have vertical direction (the parameters are therefore marked by v or V). We adapt the formulas given in [1]. The supervectors in transform domain for this field part are defined by

$$\widehat{\overline{\mathbf{H}}}^{\mathrm{v}} = \begin{bmatrix} -j\overline{\mathbf{H}}_{\mathrm{y}} \\ \overline{\mathbf{H}}_{\mathrm{x}} \end{bmatrix} \quad \widehat{\overline{\mathbf{E}}}^{\mathrm{v}} = \begin{bmatrix} \overline{\mathbf{E}}_{\mathrm{x}} \\ -j\overline{\mathbf{E}}_{\mathrm{y}} \end{bmatrix} \tag{4}$$

where the H-components are normalized with free space wave impedance $\eta_0 = \sqrt{\mu_0/\varepsilon_0}$. We assume wave propagation in y direction according to $\exp\left(-j\sqrt{\varepsilon_{re}}\overline{y}\right)$, where $\overline{y} = k_0 y$ and k_0 is the free space wave number. Using the following abbreviations [1]

$$\boldsymbol{\Lambda}^{\mathrm{v}} = \begin{bmatrix} \varepsilon_d \boldsymbol{I}_{\mathrm{h}}^{\mathrm{v}} & \widetilde{\boldsymbol{\delta}}_{\mathrm{v}}^{\mathrm{t}} \\ \widetilde{\boldsymbol{\delta}}_{\mathrm{v}} & \varepsilon_r \boldsymbol{I}_e^{\mathrm{v}} - \overline{\boldsymbol{\lambda}}_{\mathrm{ev}}^2 \end{bmatrix} \tag{5}$$

$$\overline{\boldsymbol{\delta}}_{\mathrm{v}} = \boldsymbol{T}_{\mathrm{ve}}^t\, \overline{\boldsymbol{D}}_{\mathrm{vh}}\, \boldsymbol{T}_{\mathrm{vh}} \quad \widetilde{\boldsymbol{\delta}}_{\mathrm{v}} = \sqrt{\varepsilon_{re}}\overline{\boldsymbol{\delta}}_{\mathrm{v}} \tag{6}$$

$$\boldsymbol{\Gamma}_{e,h}^{\mathrm{v}2} = \overline{\boldsymbol{\lambda}}_{e,h}^{\mathrm{v}2} - \varepsilon_d \boldsymbol{I}_{e,h}^{\mathrm{v}} \quad \widehat{\boldsymbol{\Gamma}}_{\mathrm{v}} = \mathrm{diag}\left(\boldsymbol{\Gamma}_h^{\mathrm{v}},\ \boldsymbol{\Gamma}_e^{\mathrm{v}}\right) \tag{7}$$

$$\boldsymbol{T}_{e,h}^{\mathrm{Vt}}\overline{\boldsymbol{D}}_{e,h}^t\overline{\boldsymbol{D}}_{e,h}\boldsymbol{T}_{e,h}^{\mathrm{V}} = \overline{\boldsymbol{\lambda}}_{e,h}^{\mathrm{v}2} \quad \varepsilon_d = \varepsilon_r - \varepsilon_{re} \tag{8}$$

the field transfer equations between planes A and B are given by

$$\begin{bmatrix} \widehat{\overline{\mathbf{H}}}_{\mathrm{A}}^{\mathrm{v}} \\ -\widehat{\overline{\mathbf{H}}}_{\mathrm{B}}^{\mathrm{v}} \end{bmatrix} = \underbrace{\begin{bmatrix} \overline{\boldsymbol{y}}_1^{\mathrm{v}} & \overline{\boldsymbol{y}}_2^{\mathrm{v}} \\ \overline{\boldsymbol{y}}_2^{\mathrm{v}} & \overline{\boldsymbol{y}}_1^{\mathrm{v}} \end{bmatrix}}_{\widehat{\overline{\boldsymbol{y}}}_{\mathrm{AB}}^{\mathrm{AB}}} \begin{bmatrix} \widehat{\overline{\mathbf{E}}}_{\mathrm{A}}^{\mathrm{v}} \\ \widehat{\overline{\mathbf{E}}}_{\mathrm{B}}^{\mathrm{v}} \end{bmatrix} \tag{9}$$

with

$$\begin{aligned} \overline{\boldsymbol{y}}_1^{\mathrm{v}} &= \quad \widehat{\boldsymbol{\gamma}}_{\mathrm{v}}\boldsymbol{\Lambda}_{\mathrm{v}} \ \ \widehat{\boldsymbol{\gamma}}_{\mathrm{v}} = \left(\widehat{\boldsymbol{\Gamma}}_{\mathrm{v}} \tanh(\widehat{\boldsymbol{\Gamma}}_{\mathrm{v}}\overline{d}_z)\right)^{-1} \\ \overline{\boldsymbol{y}}_2^{\mathrm{v}} &= -\widehat{\boldsymbol{\alpha}}^{\mathrm{v}}\boldsymbol{\Lambda}_{\mathrm{v}} \ \ \widehat{\boldsymbol{\alpha}}_{\mathrm{v}} = \left(\widehat{\boldsymbol{\Gamma}}_{\mathrm{v}} \sinh(\widehat{\boldsymbol{\Gamma}}_{\mathrm{v}}\overline{d}_z)\right)^{-1} \end{aligned} \tag{10}$$

where ε_r is the relative permittivity and $\overline{d}_z = k_0 d_z$ is the dimension of the region R in z direction. $\boldsymbol{I}_{e,h}^{\mathrm{v}}$ are identity matrices. $\overline{\boldsymbol{D}}_h$ ($\overline{\boldsymbol{D}}_e$) is the first order difference operator for the field component $\mathbf{H}_y(\mathbf{E}_y)$ for the line in vertical direction which has to fulfill Neumann (Dirichlet) boundary conditions. Because of the electric walls we do not get a tangential electric field at planes C and D. The tangential magnetic field there will be explained by the magnetic field vectors $\overline{\mathbf{H}}_{y\mathrm{A,B}}^{\mathrm{v}}$ and $\overline{\mathbf{H}}_{z\mathrm{A,B}}^{\mathrm{v}}$. The field at an arbitrary $\overline{z}$ ($\overline{z}_- = \overline{d}_z - \overline{z}$) is described by

$$\overline{\mathbf{H}}_u^{\mathrm{v}}(\overline{z}) = \frac{\sinh(\boldsymbol{\Gamma}_{\mathrm{w}}^{\mathrm{v}}\overline{z}_-)}{\sinh(\boldsymbol{\Gamma}_{\mathrm{w}}^{\mathrm{v}}\overline{d}_z)}\overline{\mathbf{H}}_{u\mathrm{A}}^{\mathrm{v}} + \frac{\sinh(\boldsymbol{\Gamma}_{\mathrm{w}}^{\mathrm{v}}\overline{z})}{\sinh(\boldsymbol{\Gamma}_{\mathrm{w}}^{\mathrm{v}}\overline{d}_{\mathrm{z}})}\overline{\mathbf{H}}_{u\mathrm{B}}^{\mathrm{v}} \tag{11}$$

$u \equiv y$ ($u \equiv z$) corresponds to $\mathrm{w} \equiv \mathrm{h}$ ($\mathrm{w} \equiv \mathrm{e}$). In this case the eq. (11) must be discretized for z values at the full (dashed) lines in horizontal direction. The field transfer from planes A/B to planes C/D can be described by the following equations

$$\begin{bmatrix} \overline{\mathbf{H}}_{\mathrm{uC}}^{\mathrm{v}} \\ -\overline{\mathbf{H}}_{\mathrm{uD}}^{\mathrm{v}} \end{bmatrix} = \begin{bmatrix} \overline{\boldsymbol{V}}_{\mathrm{CA}}^{\mathrm{S}} & \overline{\boldsymbol{V}}_{\mathrm{CB}}^{\mathrm{S}} \\ \overline{\boldsymbol{V}}_{\mathrm{DA}}^{\mathrm{S}} & \overline{\boldsymbol{V}}_{\mathrm{DB}}^{\mathrm{S}} \end{bmatrix} \begin{bmatrix} \overline{\mathbf{H}}_{\mathrm{uA}}^{\mathrm{v}} \\ -\overline{\mathbf{H}}_{\mathrm{uB}}^{\mathrm{v}} \end{bmatrix} \tag{12}$$

where S $\equiv$ D (on horizontal discretization lines with Dirichlet boundary conditions) if $u \equiv z$ and S $\equiv$ N (on horizontal discretization lines with Neumann boundary conditions) if $u \equiv y$ and e.g.

$$\overline{\boldsymbol{V}}_{\mathrm{CB}}^{\mathrm{S}} = \boldsymbol{T}_{\mathrm{vH}}^t\boldsymbol{\Lambda}_{\mathrm{B}}\boldsymbol{T}_{\mathrm{R}}^+ \quad \overline{\boldsymbol{V}}_{\mathrm{DB}}^{\mathrm{S}} = \boldsymbol{T}_{w\mathrm{H}}^t\boldsymbol{\Lambda}_{\mathrm{B}}\boldsymbol{T}_{\mathrm{R}}^- \tag{13}$$

$$(\boldsymbol{\Lambda}_{\mathrm{B}})_{ik} = \sinh\left(\boldsymbol{\Gamma}_{\mathrm{w}k}\overline{z}_i\right) / \sinh\left(\boldsymbol{\Gamma}_{\mathrm{w}k}\overline{d}_z\right) \tag{14}$$

The diagonal matrix $\boldsymbol{T}_{\mathrm{R}}$ obtained by

$$\boldsymbol{T}_{\mathrm{R}}^{\pm} = \mathrm{diag}\left(\sqrt{\frac{2}{N_{\mathrm{v}}}}[1/\sqrt{2}, \pm 1, 1, \pm 1, ...]\right) \tag{15}$$

is necessary to obtain the original field values at ports C (positive signs) and D (negative signs). To obtain the quantities in transform domain for the horizontal lines a multiplication by $\boldsymbol{T}_{w\mathrm{H}}^t$ is necessary. In case of $u \equiv y$ the magnetic field components on the right side of eq. (12) will be substituted by the electric field $\widehat{\overline{\mathbf{E}}}_{\mathrm{AB}}^{\mathrm{v}}$ using the eq. (9). In case of $u \equiv z$ the field vectors $\overline{\mathbf{H}}_{z\mathrm{A,B}}^{\mathrm{v}}$

as functions of $\mathbf{E}^{\mathrm{v}}_{z\mathrm{A,B}}$ have to be calculated with the help of eqs. (2.7) of [1] resulting in

$$\overline{\mathbf{H}}^{\mathrm{v}}_{\mathrm{zA,B}} = \left[-\sqrt{\varepsilon_{re}}\, \boldsymbol{I}^{\mathrm{v}}_{\mathrm{h}} \quad \overline{\boldsymbol{\delta}}^{\mathrm{t}}_{\mathrm{v}} \right] \widehat{\overline{\mathbf{E}}}^{\mathrm{v}}_{\mathrm{A,B}} \qquad (16)$$

Combining all these equations the supermatrix $\widehat{\overline{\boldsymbol{y}}}^{\mathrm{AB}}_{\mathrm{CD}}$ can be formed. The remaining matrices $\widehat{\overline{\boldsymbol{y}}}^{\mathrm{CD}}_{\mathrm{CD}}$ and $\widehat{\overline{\boldsymbol{y}}}^{\mathrm{CD}}_{\mathrm{AB}}$ can be determined in similar way. The whole analysis can now be performed as an impedance/admittance matching process [2]. The impedance/admittance matching between the regions C_1 and H_2 (note that the regions have different heights) is e.g. given by [2]

$$\overline{\boldsymbol{Z}}_{\mathrm{C1}} = \widehat{\boldsymbol{T}}^{-1}_{\mathrm{C1c}} \widehat{\boldsymbol{T}}_{\mathrm{H2}} \overline{\boldsymbol{Z}}_{\mathrm{H2}} \widehat{\boldsymbol{T}}^{-1}_{\mathrm{H2}} \widehat{\boldsymbol{T}}_{\mathrm{C1c}} \qquad (17)$$

where $\boldsymbol{T}_{\mathrm{C1c,H2}}$ are transformation matrices known in the MoL algorithm. $\boldsymbol{T}_{\mathrm{C1c}}$ is reduced according to the common part of the ports.

NUMERICAL RESULTS

The proposed algorithm is verified by comparing with other numerical methods, demonstrating precise resolution of the fields and propagation constants of the modes under consideration. It has been applied successfully to a variety of open and shielded dielectric waveguides, including the insulated image guide, which provides the most useful canonical configuration for modeling open dielectric waveguiding geometries used in millimeter through optical frequency ranges. The diagram in Fig. 4 shows the cutoff wavelength of a groove guide as function of the plate distance. For comparison the results of M. Sachidananda [6] obtained by mode matching technique (MMT) are included. Also measured results obtained by T. Nakahara and N. Kurauchi and taken form [6] are incorporated. The discretization scheme used for the calculations in this work is shown as insert in the Fig. 4. Only a quarter of the structure was used for the analysis.

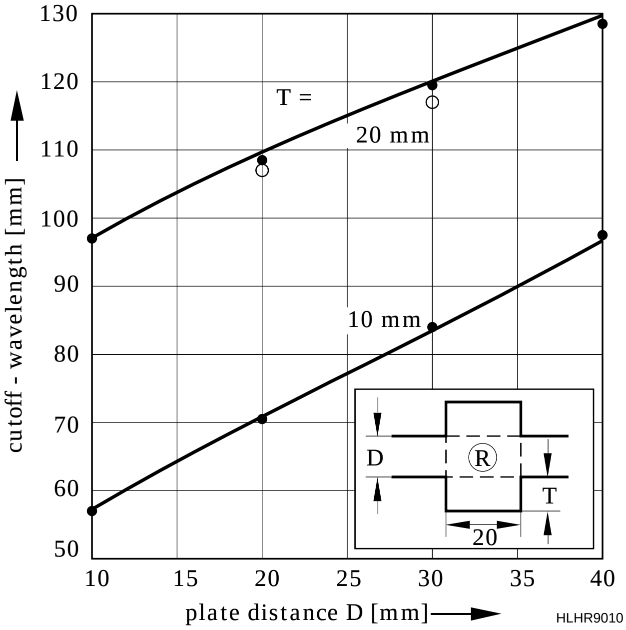

Fig. 4: Variation of the cut-off wavelength λ_c with varying plate distance of a groove guide. full line: MoL, • • • MMT [6], ◦ experiment

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